Study on Fitting Accuracy of Response Spectra for Multiple Damping Ratios

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Abstract: Employing frequency-domain methods[1] to generate seismic motions approximating the target spectrum.Simultaneously generate the power spectrum density function using the target response spectrum.The time history of acceleration can be obtained by applying an inverse Fourier transform.Nevertheless, the traditional single-damping frequency-domain method shows considerable inaccuracy in approximating multi-damping target spectra. To address this, an iterative correction technique is applied to the conventional method to achieve matched response spectra.Furthermore, the precision of the response spectrum is enhanced through the narrow-band time-history method[2].

Keywords: Narrow-band time history; Response spectrum; Seismic motion; Flat terrain

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Introduction

Earthquakes are among the most significant natural disasters affecting humankind.Consequently, from the early 20th century onwards, seismic effects have been incorporated into structural design considerations. By the mid-20th century, most national codes mandated the use of response spectrum methods for designing earthquake-resistant structures.In accordance with the requirements of the Code for Seismic Design of Buildings (GB50011-2022).In the dynamic time-history analysis of complex structures,it is necessary to employ artificial seismic motions that correspond to the design spectrum as input loads; consequently, research into methods for synthesising such artificial seismic motions holds significant practical value.Current research methodologies fall into two broad categories: time-domain methods and frequency-domain methods.Due to certain shortcomings in frequency domain fitting methods: convergence is relatively poor, particularly when fitting multiple response spectrum curves with differing damping ratios, where convergence becomes even more problematic.This paper employs a frequency-domain approximation method[3]~[5] to optimise and superimpose narrowband time histories, thereby adjusting the convergence of multi-damping response spectra to achieve closer approximation to the target spectrum.

1 Frequency Domain Approximation Method and Its Optimisation

1.1 Frequency Domain Approximation Method (Single Damping) Procedures

(1) The target spectrum is converted into a power spectrum.Kaul [6] proposed an empirical relationship in 1978 for converting response spectra into power spectra:

$$G(\omega)=-\frac{\xi}{\pi\omega\ln[-\frac{\pi}{\omega T_d}\ln(1-r)]}[S_{\alpha}^{T}(\omega)]^2 \qquad (1)$$

In Equation (1), G(ω) denotes the single-point self-power spectral density, where ω represents the angular frequency, ξ denotes the damping ratio, Td signifies the seismic motion duration, $S_{\alpha}^{T}(\omega)$ represents the target response spectrum, and r indicates the exceedance probability, typically set at 5–10%.

(2) Generate seismic waves. Determine the amplitude spectrum based on the relationship between the Fourier amplitude spectrum and the power spectrum[7]:

$$A(\omega)=\sqrt{\Delta\omega\cdot G(\omega)} \qquad (2)$$

In equation (2), $A(\omega)$ represents the Fourier magnitude, $\Delta\omega$ denotes the original frequency step, followed by an inverse Fourier transform to get the seismic time history.

(3) Generate the response spectrum. The equation of motion for a single-degree-of-freedom structure may be expressed as:

$$\ddot{Z}+2\dot{Z}\xi\omega+\omega^2 Z=-a(t) \qquad (3)$$

The response spectrum relationship is:

$$S_a(\omega_i)=\omega_i^2\bullet S_d(\omega_i) \qquad (4)$$

In Equations (3) and (4), Z 、$\dot{Z}$ 、$\ddot{Z}$ and denote the relative displacement, relative velocity, and relative acceleration of the structure, respectively. ω_i represents the displacement response spectrum at different circular frequency points, with $S_d(\omega_i)$ being the corresponding displacement response spectrum as referenced in [5].

1.2 Frequency Domain Approximation Method Optimisation

1.2.1 Inputting the Spectrum of a Multi-Damped Target[8]

Given a target spectrum $S_a^T(\omega,\xi_i)$ corresponding to m damping ratios $\xi_1,\xi_2,\dots,\xi_m$.

1.2.2 Calculation of Power Spectrum[9]

Select any damping ratio as the reference damping ratio ξ_0, with its corresponding response spectrum designated as the reference response spectrum $S_a^T(\omega,\xi_0)$. Employ formula (1) to convert the reference response spectrum into the reference power spectrum $G^{(0)}(\omega)$:

$$G^{(0)}(\omega)=\frac{\xi_0}{\pi\omega}\cdot\frac{[S_a^T(\omega,\xi_0)]^2}{-\ln[-\pi/(\omega \mathrm{T_d})\cdot\ln(1-r)]} \quad (5)$$

1.2.3 Iterative Generation of Seismic Motion[10]

Based on Equation (2), the amplitude spectrum is denoted as $A^{(k)}(\omega_c)=\sqrt{\Delta\omega\cdot G^k(\omega_c)}$, the Fourier coefficients as $F^{(k)}(\omega_c)=A^{(k)}(\omega_c)\cdot e^{i\phi_c}$, and the acceleration time history $a^{(k)}(t)$ for the kth iteration is ultimately obtained.

For each frequency component ω_c in the equation, a random phase angle ϕ_c uniformly distributed within [0, 2π] is specified (and remains unchanged).

Substitute the reference acceleration $a^{(k)}(t)$ into (3) and compute the response spectrum for all damping ratios:

$$S_a^{A,k}(\omega_n,\xi_i)=\max_t\left|\ddot{Z}(t)+\omega_n^2 Z(t)\right| \quad (6)$$

1.2.4 Determining Compliance with Requirements

Calculate the relative error between the generated spectrum and the target spectrum at each frequency point ω_n and each damping ratio ξ_i:

$$\varsigma_i^{(k)}=\left|1-R_i^k\right| \quad (7)$$

Determine whether $\varsigma_i^{(k)}$ is less than 10%. If so, output the result; if not, substitute into formula (2) and recalculate until the requirement is met.

2 Narrow Band Time Course and Its Correction

2.1 Equation of Motion

As our objective is to enhance the fitting accuracy of the frequency domain method for multiple damping ratios, we shall first employ the motion equations of a single-degree-of-freedom structure, selecting the initial seismic acceleration time history:

$$\ddot{Z}(t)+2\dot{Z}(t)\xi_k\omega_j+\omega^2 Z(t)=-a(t) \quad (8)$$

ω_j is the natural frequency of the j-th oscillator, ξ_k is the damping ratio of the k-th oscillator.

2.2 Determining the Maximum Spectral Error

Calculate the design spectrum at all frequency points within the computational timeframe; then subtract the design spectrum $S_R(\omega_j,\xi_k)$ from the target spectrum $S_\alpha^T(\omega_j,\xi_k)$.Secondly, identify the point with the largest absolute difference value and its corresponding damping ratio to determine the position corresponding to the ‘advantage’.

$$\Delta S(\omega_j,\xi_k)\max=\left|S_R(\omega_j,\xi_k)-S_\alpha^T(\omega_j,\xi_k)\right| \quad (9)$$

2.3 Pulse Function

Using the available ‘advantage’ data to construct a single-pulse function ensures the accuracy of the time-history values, as expressed by the following equation:

$$f'(t)=h_j^k(t_{mj}-\tau) \quad (10)$$

$$C'=\int_0^{t_{mj}}[h_j^k(t_{mj}-\tau)]^2 d\tau \quad (11)$$

In Equations (14) and (15), τ denotes the correction time point, t_{mj} represents the instant at which the time response at frequency ω_j attains its maximum value, and $h_j^{\,k}$ is the impulse response function.Among these,, $h_j^{\,k}(t) = -\frac{1}{\omega_{Dj}} e^{-\xi_k \omega_{Dj} t} \sin(\omega_{Dj} t)$, $\omega_{Dj} = \omega_j \sqrt{1-\xi_k^2}$. C' is the unit spectral change quantity. It can be derived using the substitution method:

Let $u = t_{mi} - \tau$, then $u = t_{mi} - \tau$. $C' = \int_u^0 [h_j^k(u)]^2 (-du) = \int_0^{t_{mi}} [h_j^k(u)]^2 du$

Using trigonometric identities: $\sin^2(\omega_{Dj} u) = \frac{1-\cos(2\omega_{Dj} u)}{2}$,

$$C' = \frac{1}{2\omega_{Dj}^2} [\int_0^{t_{mj}} e^{-2\xi_k \omega_{Dj} u} du - \int_0^{t_{mj}} e^{-2\xi_k \omega_{Dj} u} \cos(2\omega_{Dj} u) du]$$

can be obtained.

First item:

$$\int_0^{t_{mj}} e^{-2\xi_k \omega_{Dj} u} du = \frac{1 - e^{-2\xi_k \omega_{Dj} t_{mj}}}{2\xi_k \omega_{Dj}}$$

Second Item:

Let $a = 2\xi_k \omega_{Dj}, b = 2\omega_{Dj}$. Substituting into the bounds [0, tmj], we obtain

$$\int_0^{t_{mj}} e^{-au} \cos(bu) du = \frac{e^{-at_{mj}} [-a\cos(bt_{mj}) + b\sin(bt_{mj})] + a}{a^2 + b^2}$$

Perform the substitution:

$$\int_0^{t_{mj}} e^{-2\xi_k \omega_{Dj} u} \cos(2\omega_{Dj} u) du = \frac{e^{-2\xi_k \omega_{Dj} t_{mj}} [-2\xi_k \omega_{Dj} \cos(2\omega_{Dj} t_{mj}) + 2\omega_{Dj} \sin(2\omega_{Dj} t_{mj})] + 2\xi_k \omega_{Dj}}{4\omega_{Dj}^2 (\xi_k^2 + 1)}$$

To sum up:

$$C' = \frac{1}{2\omega_{Dj}^2} [\frac{1 - e^{-2\xi_k \omega_{Dj} t_{mj}}}{2\xi_k \omega_{Dj}} - \frac{e^{-2\xi_k \omega_{Dj} t_{mj}} [-2\xi_k \omega_{Dj} \cos(2\omega_{Dj} t_{mj}) + 2\omega_{Dj} \sin(2\omega_{Dj} t_{mj})] + 2\xi_k \omega_{Dj}}{4\omega_{Dj}^2 (\xi_k^2 + 1)}]$$

Through the aforementioned simplification process, the amplitude coefficient of the corrected pulse b' can be obtained:

$$b' = \frac{\Delta S(\omega_j, \xi_k)\max}{C'} \qquad (12)$$

2.4 Time History Superposition

To ensure data accuracy and prevent distortion, we introduce a single-pulse correction time scale, namely a narrow-band time scale:

$$\Delta \ddot{a}(t) = b' \cdot f'(t) \qquad (13)$$

This allows us to obtain the new schedule $\ddot{a}_{n+1}(t) = a_n(t) + \Delta \ddot{a}(t)$ .Simultaneously, calculate the response spectrum error for the new schedule to determine whether it meets the requirements.If the condition is met, output the result; if not, proceed to step 2 for recalculation.

This method employs a point-by-point correction strategy, with the advantage of modifying only one frequency point ('merit') and one damping ratio at a time, thereby avoiding distortion issues that arise from simultaneously matching multiple points.

3 Case Study

To validate the method's effectiveness, target spectra with damping ratios of 0.02, 0.03, and 0.05 were selected for fitting. The seismic motion duration was set to 20 seconds.The fundamental time history was first generated using the frequency domain iterative method described herein, followed by post-processing refinement via an optimisation approach.

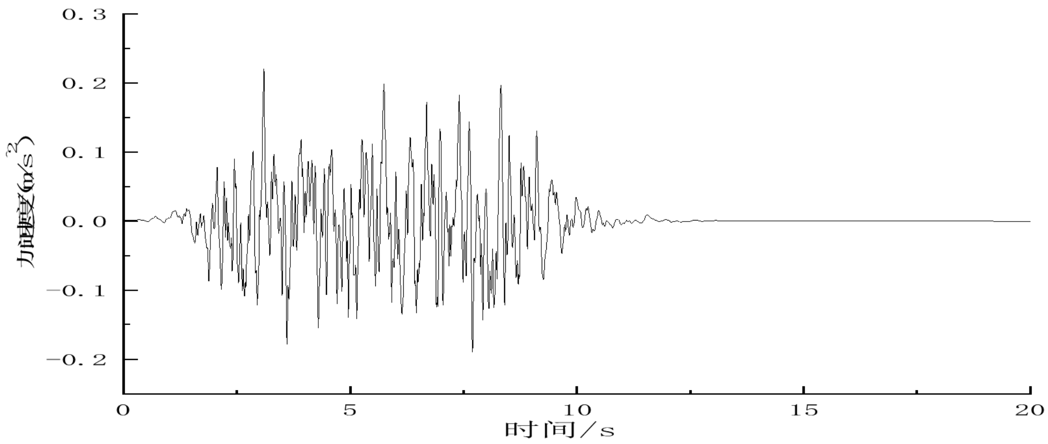

Figure 1. Time history

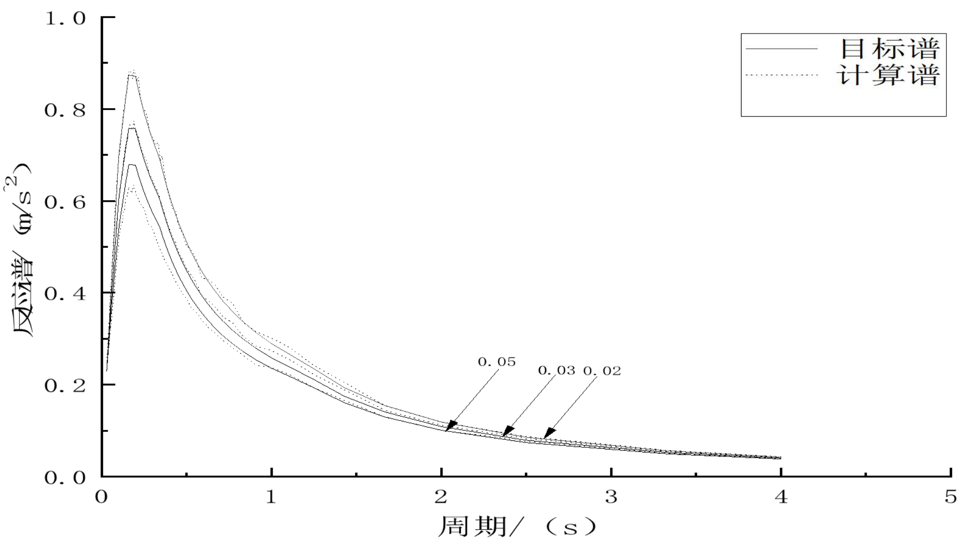

Figure 2. Comparison of response spectra

4 Summary

(1) To address the significant errors of traditional single-damping frequency-domain methods in matching target spectra with multiple damping ratios, this study introduces an iterative mechanism for optimization, effectively improving spectral matching accuracy under multi-damping conditions.

(2）From Figure 2, it can be observed that the target spectrum and the calculated spectrum are closely aligned, with an error margin of approximately 9%, which complies with the specification requirements.

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