Research on the Application of Partial Differential Equations in Engineering Fluid Mechanics

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Abstract: This paper focuses on the application of partial differential equations in engineering fluid mechanics, systematically elaborating on the theoretical basis of partial differential equations as the basic tool for describing fluid motion, and analyzing their application principles and importance in core fields of engineering fluid mechanics such as fluid flow, heat and mass transfer, and multiphase flow. Simultaneously exploring the numerical methods, analytical methods, and asymptotic methods of partial differential equations in solving engineering fluid dynamics problems, analyzing the challenges faced in the application process and proposing solutions, aiming to provide theoretical references and methodological guidance for the use of partial differential equations in solving practical problems in the field of engineering fluid dynamics, and promote the development of related engineering technologies.

Keywords: partial differential equations; Engineering fluid mechanics; Fluid motion; Numerical solution; Analytical method

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introduction

Engineering fluid mechanics is dedicated to studying the flow laws of fluids in engineering practice and their interactions with solid structures, playing a key role in many fields such as aerospace, energy and power, water conservancy engineering, and chemical engineering. Partial differential equations, with their powerful mathematical descriptive ability, have become the core tool for characterizing the fundamental laws of fluid motion. From the continuity equation, momentum equation, and energy equation that describe the macroscopic motion of fluids, to the control equation that reflects the complex physical and chemical processes of fluids, they are all presented in the form of partial differential equations. In depth research on the application of partial differential equations in engineering fluid mechanics helps to accurately understand the essence of fluid phenomena, provide theoretical basis for engineering design, optimization, and decision-making, and is of great significance for promoting engineering technology innovation and related industry development.

1 Theoretical basis of partial differential equations in engineering fluid mechanics

1.1 Deduction and expression of basic equations

The core equations of engineering fluid mechanics are constructed based on three conservation laws:

Continuity equation: Following the principle of conservation of mass, its differential form is:

$\partial t \partial \rho + \nabla \cdot (\rho v)=0$

In the equation, ρ is the fluid density, t is time, and v is the velocity vector. This equation describes the relationship between fluid density and time and space.

Momentum equation: Based on Newton's second law, the three-dimensional form is:

$\rho (\partial t \partial v + v \cdot \nabla v)=-\nabla p+ \mu \nabla 2v+ \rho g$

Among them, p is pressure, μ is dynamic viscosity, and g is gravitational acceleration. The equation reveals the dynamic relationship between force and momentum changes.

Energy equation: Following the law of conservation of energy, the general form is:

$\rho cp (\partial t \partial T + v \cdot \nabla T)=\nabla \cdot (k\nabla T)+\Phi+ST$

In the formula, cp is the specific heat capacity at constant pressure, T is the temperature, k is the thermal conductivity,

Φ is the viscous dissipation term, and ST is the heat source term, used to describe the energy conversion process.

1.2 Classification and Characteristics of Equations

According to mathematical characteristics, partial differential equations in engineering fluid mechanics can be divided into three categories (see Table 1):

Equation types, mathematical forms, physical meanings, typical application scenarios

Elliptical ∇ 2u=0 steady-state process fluid statics and steady-state seepage flow

Parabolic ∂ t ∂ u=α ∇ 2u diffusion process, heat conduction and mass diffusion

Hyperbolic ∂ t2 ∂ 2u=c2 ∇ 2u Wave Process Shock Wave Propagation and Water Hammer Phenomenon

The solution of elliptic equations depends on global boundary conditions, parabolic equations reflect the asymmetry of time and space, hyperbolic equations describe wave phenomena with characteristic lines, and different types of equations require differentiated solving strategies.

1.3 Boundary Conditions and Initial Conditions

The definite solution of partial differential equations needs to be combined with physical scene setting conditions:

Boundary conditions: reflect the interaction between fluid and boundary, such as:

No slip boundary: v · n=0 (n is the boundary normal vector), applicable to solid walls;

Free liquid surface boundary: p=p0 (p0 is atmospheric pressure), used to describe the surface of a liquid.

Initial conditions: For non-stationary problems, such as initial velocity field v (x, y, z, 0)=v0 (x, y, z), initial temperature field T (x, y, z, 0)=T0 (x, y, z), determine the initial state of the system.

Reasonable setting of conditions is the key to ensuring that the solution conforms to physical reality, and it needs to be determined comprehensively based on factors such as the geometric shape and flow state of specific engineering problems.

2 Application of Partial Differential Equations in Engineering Fluid Mechanics

2.1 Fluid Flow Issues

Partial differential equations occupy a central position in describing fluid flow problems. Whether laminar or turbulent, a system of partial differential equations composed of continuity equations and momentum equations can accurately describe the velocity and pressure field distributions of fluids. In practical engineering scenarios such as pipeline flow, channel flow, and bypass flow problems, using partial differential equations to establish mathematical models can analyze the flow characteristics of fluids, such as velocity distribution, pressure loss, resistance coefficient, etc., providing theoretical guidance for pipeline design, water conservancy engineering construction, aircraft shape optimization, etc., and achieving efficient design and performance improvement of engineering structures.

2.2 Heat and Mass Transfer Issues

Partial differential equations play an indispensable role in the field of heat and mass transfer involved in engineering fluid dynamics. The energy equation is combined with equations describing mass transfer processes to accurately depict the laws of heat transfer and material diffusion in fluids in the form of partial differential equations. Whether it is heat conduction, convective heat transfer, or mass diffusion processes, by establishing corresponding partial differential equation models, the distribution and changes of temperature and concentration fields can be studied, key parameters such as heat transfer coefficient and mass transfer rate can be analyzed, providing theoretical basis for thermal equipment design, chemical process optimization, building energy conservation, and effectively solving heat and mass transfer problems in engineering practice.

2.3 Multiphase flow problem

Multiphase flow problems involve the interaction of multiple phase fluids and complex physical processes, and partial differential equations provide powerful mathematical tools for their study. By establishing continuity equations, momentum equations, and energy equations for multiphase flow, and describing the motion laws and interactions of each phase fluid in the form of partial differential equations, the changes in phase interfaces, mass and heat transfer processes between phases, and flow characteristics in multiphase flow systems such as gas-liquid and liquid-solid can be analyzed. This provides theoretical support for equipment design and operation optimization in engineering fields such as petroleum extraction, chemical reactions, and pneumatic conveying, and promotes the development and application of multiphase flow technology.

3 Methods for Solving Partial Differential Equations in Engineering Fluid Mechanics

3.1 Numerical solution method

Numerical solution methods are commonly used to solve complex partial differential equations in engineering fluid mechanics. The finite difference method discretizes the solution region, approximates the derivative with the difference quotient, and converts the partial differential equation into an algebraic equation system for solution. It has the characteristics of intuitive principle and relatively simple programming implementation. The finite element method is based on the variational principle, dividing the solution area into a finite number of elements. Discrete equations are established by constructing element basis functions, which are suitable for dealing with complex geometric shapes and boundary conditions. It is widely used in engineering fluid mechanics. The finite volume method is based on controlling the volume, ensuring the conservation of physical quantities within each control volume. It has good conservation and physical significance in solving fluid flow and heat and mass transfer problems, and is currently one of the commonly used numerical methods in engineering calculations.

3.2 Analytical solution method

Although most practical problems in engineering fluid mechanics are difficult to obtain analytical solutions, analytical solving methods still have important value under some simplified conditions. The method of separating variables converts partial differential equations into ordinary differential equations by representing their solutions as the product of multiple univariate functions, and is suitable for solving problems with simple geometric shapes and boundary conditions. The similarity method utilizes the principle of similarity to transform partial differential equations into ordinary differential equations, which can effectively reduce the number of independent variables in the equation. It is commonly used to study problems such as boundary layer flow. Analytical solutions can clearly reveal the intrinsic relationships between physical quantities, provide theoretical basis for understanding fluid motion laws, and also serve as reference standards for verifying the accuracy of numerical methods.

3.3 asymptotic solution method

The asymptotic solution method plays an important role in dealing with problems with specific characteristics in engineering fluid mechanics. The perturbation method expands complex partial differential equations into asymptotic series form by introducing small parameters for solving problems with small deviations from ideal states. The boundary layer theory is based on the characteristic of rapid changes in fluid velocity near the solid wall, dividing the flow field into boundary layer region and external potential flow region. Different simplified equations are used to solve them, greatly simplifying the analysis process of complex flow problems and providing effective theoretical analysis methods for aircraft aerodynamic design, pipeline flow resistance analysis, etc.

4 Conclusion

Partial differential equations, as the core mathematical tool in the study of engineering fluid dynamics, play an irreplaceable role in describing fluid motion laws and solving practical engineering problems. From the establishment of theoretical foundations to the expansion in different application fields, and then to the application of diverse solving methods, partial differential equations run through the entire process of engineering fluid dynamics research. Despite facing challenges such as equation complexity and difficulty in solving in practical applications, the application of partial differential equations in engineering fluid mechanics will continue to deepen and expand with the development of mathematical theory and advances in computational technology. In the future, further strengthening the combination of theoretical research and engineering practice, exploring more efficient solution methods, will help promote technological innovation in the field of engineering fluid mechanics, and provide stronger support for the development of many industries such as aerospace, energy, and chemical engineering.

References

[1] Dai Zhimin, Li Dongping, Zhang Yu, Feng Xiaozhou Application of Partial Differential Equations in Engineering Fluid Dynamics [J]. Journal of Xi'an Technological University, 2025, 45 (03): 487-494

[2] Zhang Mengran Efficient Solution of Partial Differential Equations by Optical Neural Engine [N]. Science and Technology Daily, June 17, 2025 (004)

[3] Sun Zijian, Ma Feiyao The Homogenization Problem of Semilinear Stochastic Partial Differential Equations [J]. Journal of Mathematics, 2025, 45 (03): 225-233.

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