Finite-Time Fault-Tolerant Formation Control for Multi-UAV Systems Based on Extended State Observer

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Abstract： This paper investigates the cooperative fault-tolerant control problem for multiple unmanned aerial vehicles(UAVs) under actuator faults and external disturbances.Using a graph- theoretic framework, the communication among UAVs is struc- tured to support distributed coordination.Actuator bias faults and unknown disturbances are treated as a combined disturbance and estimated together with system states through an extended s- tate observer (ESO).To enhance estimation accuracy, an adaptive algorithm is introduced to adjust fault and disturbance parame- ters in real time.A sliding mode controller is then developed to compensate for estimation errors and actuator failures, ensuring robust tracking performance. Lyapunov-based stability analysis guarantees the finite-time convergence of formation errors under the proposed control scheme. Numerical simulations verify the effectiveness of the approach, demonstrating strong fault-tolerant coordination capabilities suitable for practical multi-UAV appli- cations.

Keywords：Fault tolerant formation control;Sliding mode controll design;Multi-UAVs.

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INTRODUCTION

WITH the increasing complexity of mission require- ments, single-agent systems face inherent limitations, such as restricted operational range and low payload capac- ity. In contrast, multi-agent cooperative control has attracted considerable attention in applications such as emergency re- sponse and collaborative reconnaissance,owing to its broader operational coverage and higher mission efficiency[1][2].Among various intelligent platforms, unmanned aerial vehicles (UAVs) stand out in cooperative control tasks and civilian rescue missions due to their compact size and relatively simple structure.As a result,research on multi-UAV coor- dination has increasingly become a focal point in the field of autonomous systems[3][4].Nevertheless, during mission execution,UAVs are often subject to unforeseen faults. If not effectively managed, these faults can significantly impair the success of the mission.This issue is particularly critical in multi-UAV cooperative operations[5][6] where the failure of a single UAV can disrupt the stability of the entire formation, potentially leading to mission failure. Therefore, developing robust fault-tolerant cooperative control strategies is essential to ensure the reliability and continuity of multi-UAV missions.

Coordinated control of multi-UAV systems is a critical re- search frontier, with applications in surveillance,environmen- tal monitoring,search and rescue,and logistics.Challenges include nonlinear dynamics,disturbances,limited communi- cation,actuator saturation, and environmental uncertainties. Recent advances integrate adaptive control, event-triggered mechanisms, robust estimation, and distributed optimization to enhance reliability and performance. Recent studies have shown that adaptive distributed control architectures can ef- fectively compensate for compound system uncertainties, en- abling robust tracking performance while relying only on local and neighboring UAV information[7][8].Finite-time and fixed-time control frameworks, incorporating nonlinear mapping or predictor-based strategies, have been proposed to guarantee fast convergence despite input saturation, environ- mental disturbances, or system constraints [9]-[11].

Furthermore, the introduction of advanced observer design- s,such as finite-time disturbance observers (FTDOBs) and extended state observers (ESOs), has enhanced the system’s resilience against exogenous disturbances and measurement noise, ensuring precise compensation in both centralized and distributed configurations [12]-[14]. Beyond stability and ro- bustness, the multi-UAV formation problem increasingly de- mands efficient resource utilization, particularly under commu- nication and computation limitations. Event-triggered control schemes, leveraging adaptive sliding mode or terminal sliding mode designs, have been developed to significantly reduce control update frequencies while ensuring fixed-time stability and avoiding Zeno behavior [8][14]. In addition, distributed formation control strategies that account for moving targets, time-varying environmental factors or dynamically partitioned subgroups have broadened the application scope of multi- UAV formations, supporting complex mission execution under dynamic conditions [15][16]. Despite these advances, several open issues persist, including managing full-state constraints, overcoming the explosion of complexity associated with non- linear transformations, and ensuring scalability for large-scale UAV swarms. Addressing these challenges requires the contin- uous integration of control-theoretic innovations, optimization frameworks, and advanced sensing and communication proto- cols [17].

In most existing studies, actuator faults are widely rec- ognized as a predominant factor contributing to formation failures in both homogeneous and heterogeneous multi-agent systems. These faults may trigger abrupt structural pertur- bations, posing substantial challenges to the system's abil-

ity to preserve its intended configuration. To address these challenges, various distributed control frameworks have been proposed to enable UAVs to collaboratively achieve formation objectives under uncertain conditions. Notably, the integra- tion of fuzzy logic systems with backstepping and dynamic surface control techniques has enhanced the capacity to deal with unknown nonlinearities and actuator anomalies, while ensuring collision avoidance and connectivity preservation within dynamic formations [18]. Fractional-order and fixed- time control paradigms have also been leveraged to improve convergence rates and robustness against state constraints, par- ticularly when operating under asymmetric and time-varying bounds [19][20]. Moreover, observer-based methods, such as extended state observers and super-twisting sliding mode observers, have proven effective in estimating lumped distur- bances and compensating for sensor unavailability and actuator degradations in real time [21]-[23]. These techniques are fur- ther complemented by event-triggered mechanisms and neural network approximators, which reduce computational overhead and enhance online adaptability to uncertain flight conditions [19][22]. Despite these advances, achieving guaranteed per- formance in the presence of multiple concurrent faults and maintaining formation

integrity during abrupt maneuvers or trajectory transitions remain open research problems. There- fore, the ongoing evolution of FTFC strategies continues to emphasize distributed intelligence, adaptive estimation, and resilient nonlinear control, laying the foundation for highly autonomous and reliable multi-UAV systems [24]-[27].

Inspired by recent studies, this work addresses fault-tolerant cooperative control for multi-UAV systems under simultaneous actuator faults and external disturbances by leveraging graph- theoretic neighbor information and employing an extended state observer with adaptive estimation. A sliding mode con- troller is developed to compensate for estimation errors and actuator failures, with Lyapunov-based analysis guaranteeing finite-time tracking performance, numerical simulations verify the effectiveness of the proposed FTFC scheme.

1 PRELIMINARIES

1.2 Graph Theory

Considering the MAS, an undirected, connected topolo- gy graph $\mathcal{G} = \{\mathcal{V}, \varepsilon, \mathcal{A}\}$, where $\mathcal{V} = \{n_1, n_2, \ldots, n_N\}$, $\varepsilon \subseteq \mathcal{V} \times \mathcal{V} = \{n_i, n_j . i, j \in \mathcal{V}\}$, and $\mathcal{A} = \{a_{ij} \in \mathbb{R}^{N \times N}\}$ respectively denote the node set, edge set, and adjacency matrix. If there exists an information communication from n_j and n_i, it implies that n_i is a neighbor of n_j. The element a_{ij} in the adjacency matrix is set as a positive value if the *ith* UAV can obtain the information from the *jth* UAV, otherwise, $a_{ij} = 0$. Define the degree matrix $\mathcal{D} = diag\{d_1, d_2, \ldots, d_N\}$. Subsequently, the Laplacian matrix $\mathcal{L}$ is defined as

$$\mathcal{L} = \mathcal{D} - \mathcal{A} .$$

By augmenting the graph $\mathcal{G}$ with one leader UAV, the leader adjacency matrix is defined as $B = diag\{b_1, b_2, ..., b_N\}$, and the element bi is positive if the ith UAV has access to the leader UAV; otherwise, $b_i = 0$.

1.3 Definitions and Lemmas

Proposition 1: The graph G is undirected and connected.

Lemma 2: [28] The Laplacian matrix with respect to an undirected connected graph is irreducible.

Lemma 3: [28] If the Laplacian matrix L is irreducible, then $L + B$ is a positive definite matrix.

Definition 4: With regard to consensus control in leader- follower multi-UAV systems, the following conditions must be satisfied,

$\lim \|z_i(t)\| = \lim \|x_i(t) - x_{di}(t) - \eta_i\| = 0,$

i = 1, 2, . . . , N

where $\eta_i = [\eta_{i1} \quad \eta_{i2} \quad ... \quad \eta_{im}]^T \in R^m$ represent the relative position vector between the ith follower agent and the leader, which defines the desired formation geometry. The formation's reference trajectory is specified by a time-varying signal $x_{di}(t) \in R^m$, where $x_{di}(t)$ both and its time derivative

$\dot{x}_{di}(t)$ are assumed to be bounded.

The following definitions and lemmas will be utilized in the algorithm design.

Lemma 5: [29] For any positive constants $r1, r2$ and ι, there exist real variables p and q satisfying the following inequality:

$$|p|^{r1}|q|^{r2} \le \frac{r1}{r1+r2}\iota|p|^{r1+r2} + \frac{r2}{r1+r2}\iota^{\frac{-r1}{r2}}|q|^{r1+r2}.$$

Lemma 6: [30] Define a real n-dimensional vector $\omega = [\omega_1, \omega_2, ...\omega_n]^T \in \mathbb{R}^N$ that satisfies $\|\omega\|_2 = \sqrt{\sum_{i=1}^{N}|\omega_i|^2} \equiv \sqrt{\omega^T\omega}.$

2 MAIN RESULTS

2.1 Dynamic Mode

A multi-UAV system with N follower agents is considered, where the dynamics of each follower are described by:

$$\begin{cases} \dot{p}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) + d_i(t) \end{cases}, \qquad (1)$$

where $i = 1, 2, ..., N$, $p_i(t) = \left[x_i(t) y_i(t)\right]^T$, $v_i(t) = \left[v_{xi}(t) v_{yi}(t)\right]^T$, $v_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^n$ represent the velocity, and control inputs, respectively. $d_i(t)$ represents the external disturbance acting on the UAV.

The failure model described by the following equation

$u_i(t) = \rho u_{fi}(t) + u_{di}(t)$ (2)

where ρ_i denotes the actuator efficiency factor, which satisfies $0 \le \rho(t) \le 1$, and $u_{di}(t)$ represents an unknown bounded bias fault. The expression (2) includes both normal and faulty scenarios:

1) Healthy actuators: $\rho(t) = 0$ and $d_{i3}(t) = 0$;
2) Partial actuator faults (loss of effectiveness): $0 < \rho < 1$ and $d_{i3}(t) = 0$.

The estimates of $D_i(t)$ and $\rho_i(t)$ are denoted by $\hat{D}_i(t)$ and $\hat{\rho}_i(t)$,respectively.~Consequently, the corresponding estimation errors $\tilde{D}_i(t)$ and $\tilde{\rho}_i(t)$ are given by:

$\tilde{D}_i(t) = \hat{D}_i(t) - D_i(t)$ (3)

$\tilde{\rho} = \hat{\rho} - \rho$ (4)

Assumption 3: The lumped disturbance estimation error is

bounded by $E_D = \sup_{t>0}|\hat{D}_i(t)|$.

Substituting (2) into (1) yields

$$\begin{cases} \dot{p}_i(t) = v_i(t) \\ \dot{v}_i(t) = \rho u_{fi}(t) + D_i(t) \end{cases}, \quad (5)$$

where $D_i(t) = u_{di}(t) + d_i(t)$ represents the lumped uncer- tainties.

The leader UAV's dynamics in the formation system are given by:

$$\begin{cases} \dot{p}_0(t) = v_0(t) \\ \dot{v}_0(t) = f_{ref}(t) \end{cases}, \quad (6)$$

where $p_0(t) = \left[x_0(t)\, y_0(t)\right]^T$, $v_0(t) = \left[v_{x0}(t)\, v_{y0}(t)\right]^T$ and $f_{ref}(t)$ represent a continuous function that describes the reference trajectory.

2.2 Extended State Observer Design

In large-scale multi-UAV formation control systems, strong coupling effects often exist among the states of individual UAVs. As the formation size increases, it becomes increasingly difficult to directly measure certain state variables of some UAVs. To address this issue of limited observability, an ex- tended state observer is designed to estimate the unmeasurable states, enabling more effective system monitoring and control.

According to (5), the *ith* follower UAV's position $p_i(t)$, ve- locity $v_i(t)$ and the lumped disturbance $D_I(t)$ are represented by the state variables $x_{i1}(t), x_{i2}(t), x_{i3}(t)$, respectively.

Consequently, the dynamic model of the ith follower UAV can be expressed as:

$\dot{x}_{i2}(t) = \rho u_{fi}(t) + D_i(t)$ (7)

By treating the lumped disturbance of the system as an augmented system state, (7) can be rewritten as:

$$\begin{cases} \dot{x}_{i1}(t) = x_{i2}(t) \\ \dot{x}_{i2}(t) = x_{i3}(t) + \rho u_{i1}(t) \\ \dot{x}_{i3}(t) = \eta_i(t) \\ y_{ip}(t) = x_{i1}(t) \end{cases} \quad (8)$$

$\eta_i(t)$ is a bounded smooth function, i.e., there exists a positive number η0 that satisfies $|\eta_i(t)| < \eta_0$.

Proposition 7: The state $x_{i2}(t)$ is unknown and only the system output $y_{ip}(t)$ is observable.

The design of an extended state observer is proposed for the multi-UAV formation control systems:

$$\begin{cases} \dot{\hat{x}}_{i1}(t) = \hat{x}_{i2}(t) + \gamma_{i1}\delta_i\left(x_{i1}(t) - \hat{x}_{i1}(t)\right) \\ \dot{\hat{x}}_{i2}(t) = \hat{x}_{i3}(t) + \rho u_{i1}(t) + \gamma_{i2}\delta_i^2\left(x_{i1}(t) - \hat{x}_{i1}(t)\right) \\ \dot{\hat{x}}_{i3}(t) = +\gamma_{i3}\delta_i^3\left(x_{i1}(t) - \hat{x}_{i1}(t)\right) \end{cases} \quad (9)$$

where, $\hat{x}_{i1}(t), \hat{x}_{i2}(t)$ and $\hat{x}_{i3}(t)$ represent the estimate value of $x_{i1}(t), x_{i2}(t)$ and $x_{i3}(t)$ respectively. $\gamma_i = [\gamma_{i1} \quad \gamma_{i2} \quad \gamma_{i3}]$

is the gain vector, $\gamma_{ij} > 0, j = 1,2,3 . \delta_i > 1$ is the gain

parameter.

Define the observer error and scaled error as

$$\delta_i = x_i - \hat{x}_i$$

$$\varepsilon_{ij}(t) = \tilde{x}_{ij}(t)\delta_i^{3-j}$$

The dynamic model of the scaled error is expressed as follows:

$\dot{\varepsilon}_i(t) = A_i\varepsilon_i(t) + I_0\eta_i(t)$ (10)

where, $\varepsilon_{ij}(t) = \left[\varepsilon_{i1}(t)\varepsilon_{i2}(t)\varepsilon_{i3}(t)\right]^T, I_0 = [0 \quad 0 \quad 1]^T,$ and the

matrix $A_i = \left| -\gamma_{i2}\delta_i \quad 0 \quad \delta_i \right|$ is the Hurwitz matrix.

C. Sliding Mode Controler Design

According to Definition 2, the following error dynamics can be generated:

$$\begin{cases} z_{i1}(t) = \hat{p}_i(t) - p_{ir}(t) \\ z_{i2}(t) = \hat{v}_i(t) - v_{ir}(t) \end{cases} \quad (11)$$

where, $p_{ir}(t) = p_0(t) + h_{ip}(t)$ and $v_{ir}(t) = v_0(t) + h_{iv}(t)$. $\hat{p}_i(t)$ and $\hat{v}_i(t)$ represent the estimated value of $p_i(t)$ and $v_i(t)$ respectively.

The global tracking error of the formation system is con- structed based on the observer state information with (9):

$$\begin{cases} e_{ip}(t) = \sum_{j\in N_i} a_{ij}\left(z_{i1}(t) - z_{j1}(t) + g_i z_{i1}(t)\right) \\ e_{iv}(t) = \sum_{j\in N_i} a_{ij}\left(z_{i2}(t) - z_{j2}(t) + g_i z_{i2}(t)\right) \end{cases} \quad (12)$$

Based on the designed tracking error vector, the sliding mode surface for the ith follower UAV is defined as follows:

$s_i(t) = \mu e_{ip}(t) + e_{iv}(t), i = 1,2,...,N, \mu > 0.$ (13)

According to [19], proper tuning of the observer parame- ters guarantees that the observation error remains sufficiently

small. Consequently, $\hat{x}_{i2}(t)$ can effectively substitute $\dot{p}_i(t)$,

while $\hat{x}_{i3}(t)$ can be used in place of the lumped disturbance

$D_i(t)$. Taking the time derivative of (13) yields the following expression:

$= \mu e_{iv}(t) + N_i\left(D_i(t) + \rho(t)u_{i1}(t) + \gamma_{i2}\delta_i^2\left(x_{i1}(t) - \hat{x}_{i1}(t) - \dot{h}_{iv}(t)\right)\right)$ (14)

where, $N_i = \left| \sum a_{ij} + g_i \right|$, $\Xi_i(t) =$

$-\sum a_{ij}\left(\dot{v}_{ir}(t) - \dot{h}_{iv}(t)\right) - g_i \dot{v}_0(t)$

Let $\lambda = \frac{1}{}$,where $\hat{\lambda}$ denotes the estimate of λ ,and $\tilde{\lambda}$

represents the estimation error, we can obtain:

$\tilde{\lambda} = \lambda - \hat{\lambda}$ (15)

The sliding mode control input ui (t) is designed as follows:

$u_i(t) = -N_i^{-1}\hat{\lambda}\left(k_1 s + k_2 sign(s) + N_i A + \mu e_{iv}(t) + \Xi_i(t)\right)$ (16)

where $sign(s_i(t))$ is a sign function, $\omega > 0, \xi > 0$ represents the control gain, and $\omega > 3N\psi$

.

The adaptive law $\hat{\Psi}_i(t)$ can be designed as:

$\dot{\hat{\lambda}} = s\gamma sign(\rho)(k_1 s + k_2 sign(s) + N_i A + \mu e_{iv}(t) + \Xi_i(t))$ (17)

3 STABILITY ANALYSIS

Theorem 8: Considering the multi-UAV formations system with actuator faults and lumped disturbances, when Assump- tion 2 and Assumption 3 hold, by introducing the extended state observer (9) and given the designed sliding mode surface (13) and control input (16) guarantees that the position error and observer estimation error converge to zero in finite-time.

Proof: To verify the convergence of the designed extend- ed state observer, the following Lyapunov candidate function is constructed:

$V_{i0} = \varepsilon_i^T P_i \varepsilon_u$ (18)

where, , $P_i = P_i^T > 0, P_i A_i + A_i^T P_i = -I_i$, I_i is the

compatible identity matrix.

According to Lemma 2, the time derivative of (18) is obtained as follows:

$\leq -\varepsilon_i^T(t)\varepsilon_i(t) + \frac{1}{4}\|\varepsilon_i(t)\|_2^2 + 4\|P_i\|_2^2 I_\vartheta \eta_i^2$ (19)

where, $\lambda_i = \frac{3}{4\lambda_{\max}(P_i)}$. Let $p = 1, q = \varepsilon_i^T P_i \varepsilon_I, r_1 = 1 - l,$

$r_2 = l, \iota = 1$ based on Lemma 1, the following inequality is obtained:

$\left(\varepsilon_i^T(t) P_i \varepsilon_i(t)\right)^l \leq 1 - l + l\varepsilon_i^T(t) P_i \varepsilon_i(t)$ (20)

Based on (20), (19) can be rewritten as:

$+\lambda_i(1-l) + 4\lambda_{\max}\left(P_i^T P_i\right)\eta_{i0}^2$ (21)

where, $g_{i1}=\lambda_i\left(1-l\right), g_{i2}=\lambda_i, \Lambda_i=\lambda_i\left(1-l\right)+4\lambda_{\max}P_i^T\eta_{i0}^2.$

Proof: In order to analyze the convergence properties of the sliding mode control, the following Lyapunov candidate function is considered:

$V_{i1}=\sum^{N}\frac{1}{2}s_i^2+\frac{|\rho|}{2}\tilde{\lambda}^2$ (22)

Taking the time derivative of (22) and substituting (16) and (17) yields:

$$\begin{aligned}\dot{V}_{i1}&=\sum_{i=1}^{N}s\left(\begin{array}{c}\mu e_{iv}\left(t\right)-N_i\rho\hat{\lambda}N_i^{-1}\left(\begin{array}{c}k_1s+k_2sign\left(s\right)\\+N_iA+\mu e_{iv}\left(t\right)+\Xi_i\left(t\right)\end{array}\right)\\+N_iA+\Xi_i\left(t\right)\end{array}\right)\\&-\frac{|\rho|}{\gamma}\tilde{\lambda}\left(s\gamma sign\left(\rho\right)\left(k_1s+k_2sign\left(s\right)+A+N_i^{-1}\Xi_i\left(t\right)\right)\right)\\&=\sum_{i=1}^{N}s\left(\begin{array}{c}\mu e_{iv}\left(t\right)-\rho\hat{\lambda}N\left(\begin{array}{c}k_1s+k_2sign\left(s\right)+N_iA+\mu e_{iv}\left(t\right)\\+\Xi_i\left(t\right)\end{array}\right)\\+N_iA+\Xi_i\left(t\right)\end{array}\right)\\&-s\rho\tilde{\lambda}\left(\begin{array}{c}k_1s+k_2sign\left(s\right)+N_iA+\mu e_{iv}\left(t\right)\\+\Xi_i\left(t\right)\end{array}\right)\\&=\sum_{i=1}^{N}-k_1s^2-k_2\left|s\right|\\&\le 0\end{aligned}\qquad(23)$$

According to Lemma 3, (23) satisfies the following inequal- ity:

$\dot{V}_{i1}\le-\alpha_1V_{i1}^{\frac{1}{2}}-\alpha_2V_{i1}$ (24)

To demonstrate the global stability of the system, the Lyapunov candidate function is selected as:

$V_p=\sum^{N}V_{i0}+V_{i1}$ (25)

Taking the time derivative of (25) and substituting (21) and

(24) yields:

$\dot{V}_p\le-C_1V_p-C_2V_p^l+M$ (26)

where, $C_1=\min\left[g_{i1},\alpha_2\right], C_2=\min\left[g_{i2},\alpha_1\right], M=\sum_{i=1}^{N}\Lambda_i$

Based on Lemma 4 and (??), it can be concluded that the error converges in finite time, and the convergence time Ts satisfies:

$$T_s\le\max\left\{\begin{array}{c}\frac{1}{\theta C_1\left(1-l\right)}\ln\frac{\theta C_1V_P^{1-l}\left(t_0\right)+C_2}{C_2}\\\frac{1}{C_2\left(1-l\right)}\ln\frac{C_1V_P^{1-l}\left(t_0\right)+\theta C_2}{\theta C_2}\end{array}\right\}\qquad(27)$$

where, $\theta\in\left(0,1\right)$.

In summary, the proposed fault-tolerant control algorithm ensures finite-time convergence for multi-UAV formation sys- tems.

4 NUMERICAL SIMULATION

This section presents numerical simulations to verify the effectiveness of the proposed control algorithm. The multi- UAV formation system comprises six UAVs, including one leader (UAV0) and five followers (UAV1–UAV5). The forma- tion vector for the ith follower UAV is defined as:

$$H_i\left(t\right)=\begin{bmatrix}H_{ipx}\\H_{ipy}\\H_{ivx}\\H_{ivy}\end{bmatrix}=\begin{bmatrix}2\cos\left(0.5t+2\pi\left(i-1\right)/5\right)\\2\sin\left(0.5t+2\pi\left(i-1\right)/5\right)\\-\sin\left(0.5t+2\pi\left(i-1\right)/5\right)\\\cos\left(0.5t+2\pi\left(i-1\right)/5\right)\end{bmatrix}$$

The initial positions of the follower UAVs are set to $[-2\quad -1]^T,[-2\quad 0]^T,[0\quad 1]^T,[2\quad 1]^T,[2\quad -1]^T$ respectively, while the leader UAV0 is initialized at $\begin{bmatrix}0 & 0\end{bmatrix}^T$. The position trajectory and control input for the leader UAV are given by $p_0(t)=\begin{bmatrix}0.1t & 0.1t\end{bmatrix}^T$. Also design the lumped disturbance as $D_i(t)=0.1\sin(0.3p_i(t))+0.1\sin(0.3v_i(t))$. Select the above designed parameters as $\mu=1.85, k_1=25, k_2=0.25,$ $\gamma=1.2, k_{11}=15, k_{12}=15, k_{13}=25, k_{21}=9, k_{22}=13,$ $k_{23}=20, k_{31}=8, k_{32}=15, k_{33}=20, k_{41}=10, k_{42}=15,$ $k_{43}=20, k_{51}=15, k_{52}=15, k_{53}=10, r_1=1, r_2=2$ $r_3=4, r_4=4, r_5=4.5$. Figs.1-Figs.8 show the simulation results.

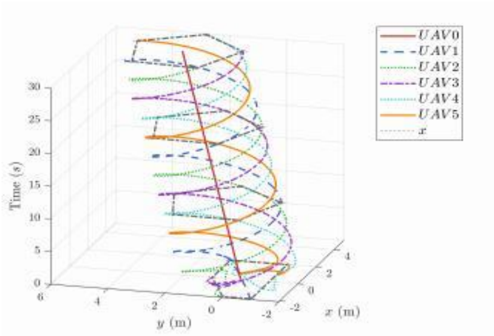

Fig. 1. Multi-UAVs formation flight trajectories.

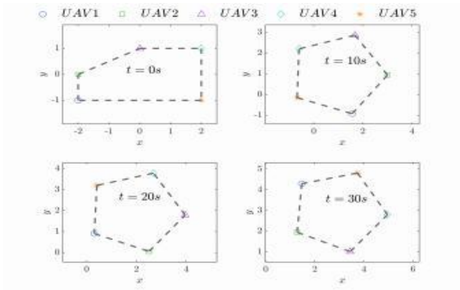

Fig. 2. Multi-UAVs formation flight trajectories.

Fig.1 and Fig.2 demonstrate the formation flight trajectory, confirming the accurate tracking performance of the proposed method. Fig.3 and Fig.4 present the position tracking errors along the X-axis and Y-axis, demonstrating convergence to a small neighborhood near zero within 2 seconds. Figs.5 and Figs.6 demonstrate the velocity tracking errors of the UAVs. Despite actuator faults, the five UAVs maintain velocity consensus and achieve cooperative formation flight. Figs.7 and Figs.8 depict the error curves between the observed and actual

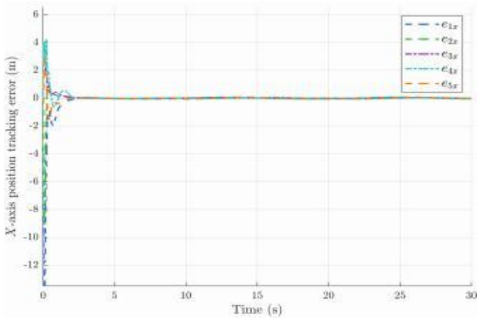

Fig. 3. Position Tracking Error.

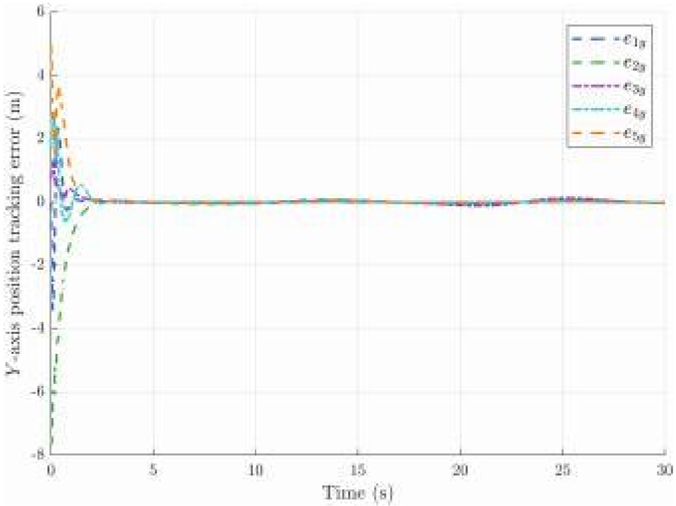

Fig. 4. Tracking Error.

position values for the X-axis and Y-axis, respectively.

5 CONCLUSION

This paper presents a cooperative fault-tolerant control strategy for multi-UAV systems under actuator bias faults and external disturbances. By modeling the UAV communication topology with graph theory, an extended state observer is designed to simultaneously estimate unknown system states and lumped disturbances. An adaptive sliding mode controller is developed to compensate for actuator failures and estimation residuals, ensuring robust formation tracking. Lyapunov-based analysis proves the finite-time convergence of the proposed method. Numerical simulations validate its effectiveness in maintaining formation coordination despite faults and disturbances. Future work will explore fault-tolerant control under switching topologies and more complex disturbance environments to further enhance system resilience.

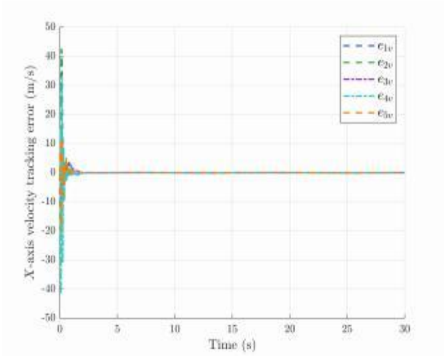

Fig. 5. Velocity Tracking Error.

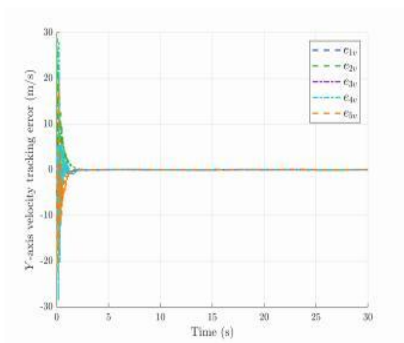

Fig. 6. Velocity Tracking Error.

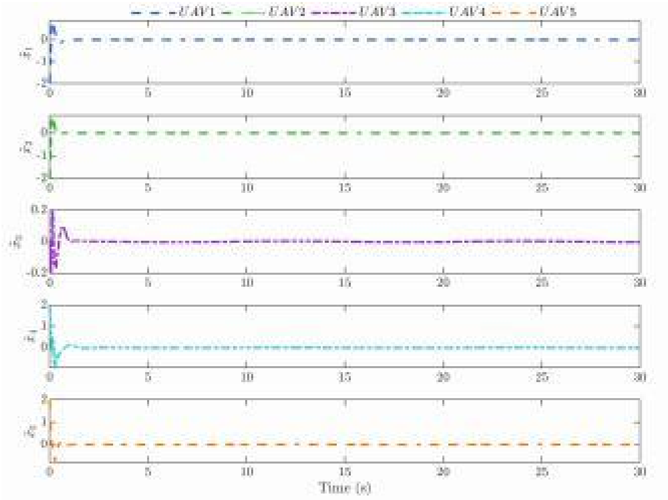

Fig. 7. Estimation error.

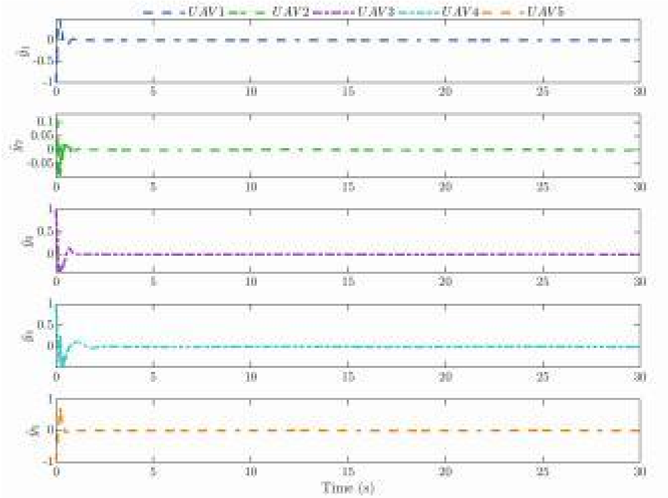

Fig. 8. Estimation error.

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